Towards the QCD phase diagram at finite chemical potential

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for the Wuppertal-Budapest-Collaboration

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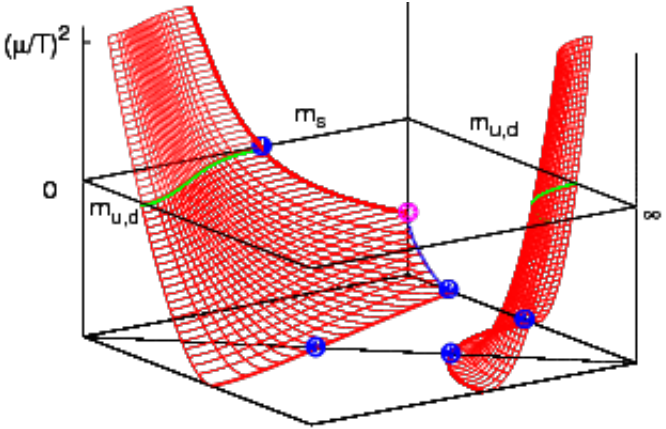
SFB TR55

BERGISCHE UNIVERSITÄT WUPPERTAL

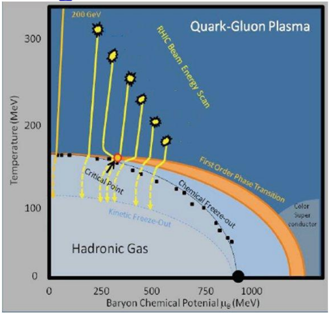
WB collaboration

The QCD phase diagram



sorce: C. Bonati et al. axXiv:1311.0473 (2014)

The (T, μ_B)-phase diagram of QCD



Our observables:

Last Year: T_c

R. Bellwied et al., Phys. Lett. B751, 559 (2015), arXiv:1507.07510

This year: The Equation of State

J. Günther et al., arXiv:1607.02493

The sign problem

The QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle- antiparticle-symmetry $\det M(U)$ is real
- If $\mu > 0$ $\det M(U)$ is complex

Dealing with the sign problem

- Reweighting technics
- Canonical ensemble $\rightarrow$ 14:20 Vitaly Bornyakov
- Complex Langevin $\rightarrow$ 14:50 Benjamin Jäger
- Density of state methods
- Dual veriables
- Taylor expansion
- Imaginary μ
- ...

The Taylor expansion method

The pressure can be written as:

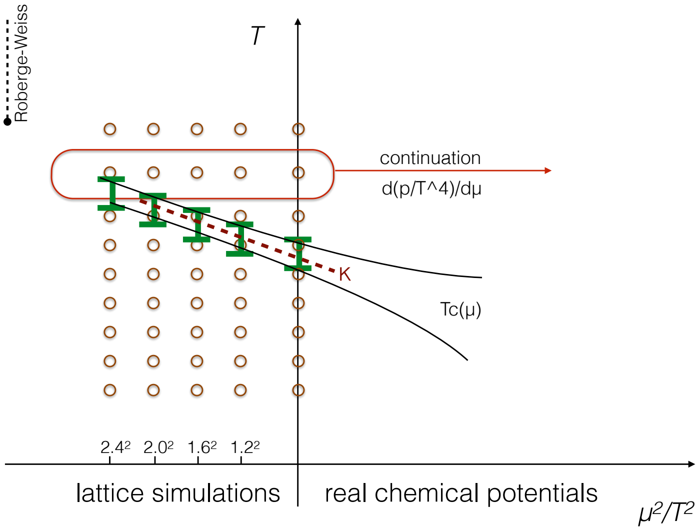
$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

with $X = B, Q, S$: conserved charges
χ_n^X can be determined on the lattice at $\mu = 0$ as:

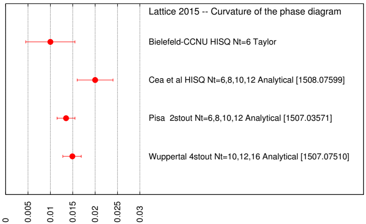
$$\chi_n^X = \frac{\partial^n \left(\frac{P}{T^4}\right)}{\partial \left(\frac{\mu_X}{T}\right)^n}$$

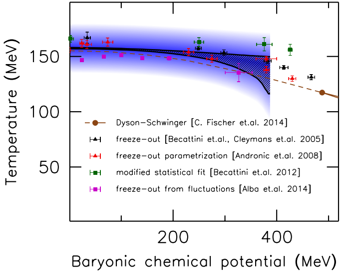
With the Taylor coefficients the observables can be extrapolated to finite chemical potentials

Imaginary μ

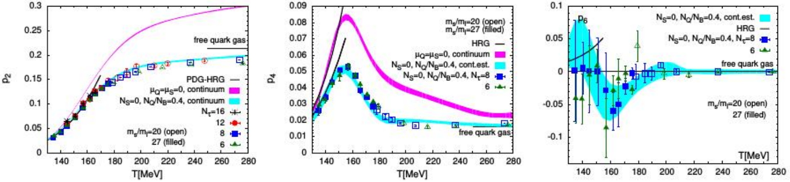


T_c

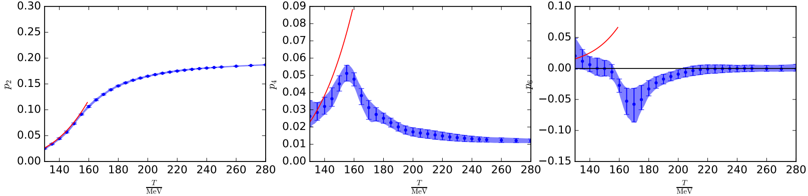

Lattice 2015 -- Curvature of the phase diagram
Bielefeld-CCNU HISQ Nt=6 Taylor
Cea et al HISQ Nt=6,8,10,12 Analytical [1508.07599]
Pisa 2stout Nt=6,8,10,12 Analytical [1507.03571]
Wuppertal 4stout Nt=10,12,16 Analytical [1507.07510]
0
0.005
0.01
0.015
0.02
0.025
0.03


Temperature (MeV)
150
100
50
200
400
Baryonic chemical potential (MeV)
Dyson−Schwinger [C. Fischer et.al. 2014]
freeze−out [Becattini et.al., Cleymans et.al. 2005]
freeze−out parametrization [Andronic et.al. 2008]
modified statistical fit [Becattini et.al. 2012]
freeze−out from fluctuations [Alba et.al. 2014]

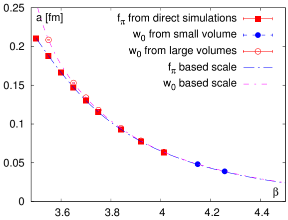
The Equation of State



sorce: Talk of C. Schmidt at Conf2016



Simulation details



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, in contrast to simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5$
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Observables

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial (m_q)^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right)\frac{m_l^2}{m_\pi^4}$$

Chiral condensate:

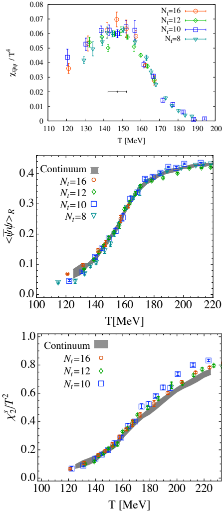
$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

$$\langle\bar{\psi}\psi\rangle^r = -\left(\langle\bar{\psi}\psi\rangle(T,\beta) - \langle\bar{\psi}\psi\rangle(0,\beta)\right)\frac{m_l}{m_\pi^4}$$

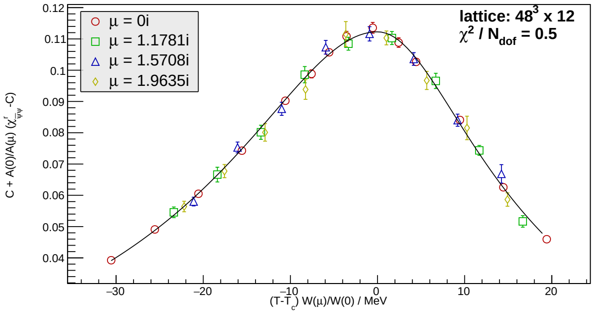
Strangeness susceptibility:

$$\chi_{ss} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial (\mu_s)^2}$$

S. Borsányi et al (2010, arXiv:1005.3508)



$\chi_{\bar{\psi}\psi}$

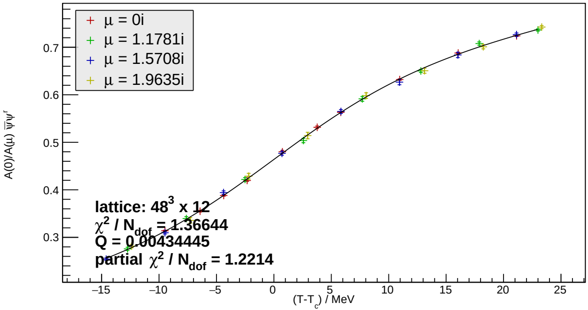


Fit function:

$$\chi^r_{\bar{\psi}\psi}(T) = \begin{cases} C + A^2(\mu)\left(1 + W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T \le T_c \\ C + A^2(\mu)\left(1 + b^2 W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T > T_c \end{cases}$$

(or $\chi^r_{\bar{\psi}\psi}(T) = C + \frac{A(\mu)}{1+W^2(\mu)(T-T_c(\mu))^2+a_3 W^3(\mu)(T-T_c(\mu))^3}$)

$\bar{\psi}\psi$

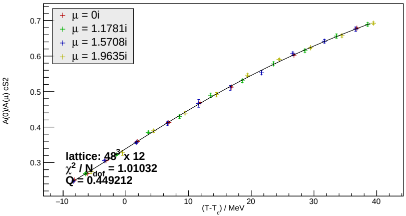


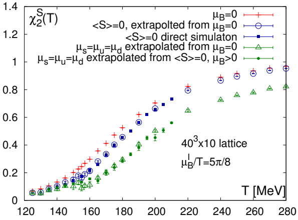
Fit function:

$$\langle\bar{\psi}\psi\rangle^r(\mu, T) = A(\mu)\left(1 + B\tanh\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$$

$$\left(\text{ or } \bar{\psi}\psi^r(\mu, T) = A(\mu)\left(1 + B\arctan\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)\right)$$

χ_{SS}



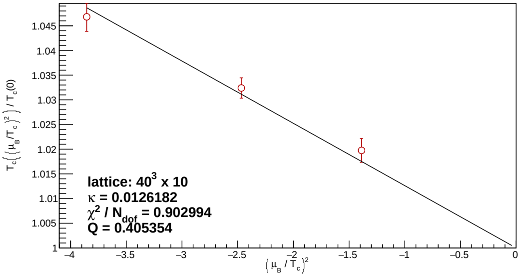


Fit function:

$$\chi_{SS}(\mu, T) = A(\mu)\left(1 + B \tanh\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$$

(or $\chi_{SS}(\mu, T) = A(\mu)\left(1 + B \arctan\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$)

Curvature



Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

For error analysis we also fit:

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Continuum extrapolation

Continuum extrapolation:

$$\kappa = \kappa^{\mathrm{c}} + A\left(\frac{1}{N_t}\right)^2$$

Combined curvature fit and continuum extrapolation with:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{\mathrm{c}} + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$$

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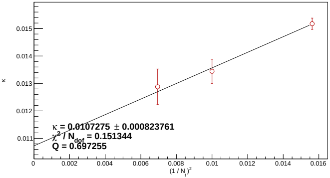
Extrap. with $Nt = 8, 10, 12$



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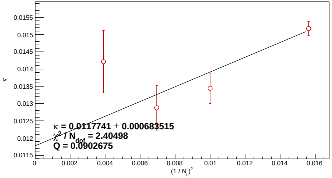
Extrap. with $Nt = 8, 10, 12, 16$



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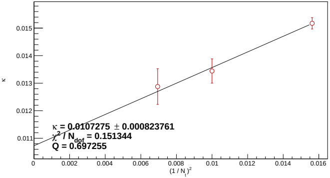

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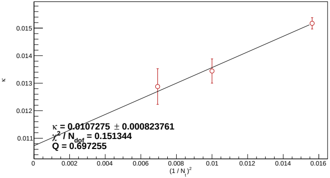

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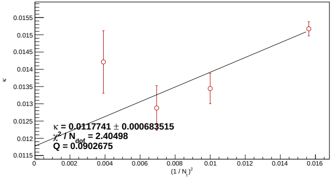
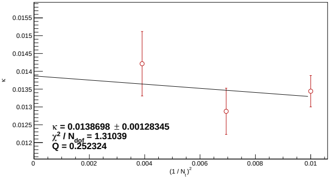
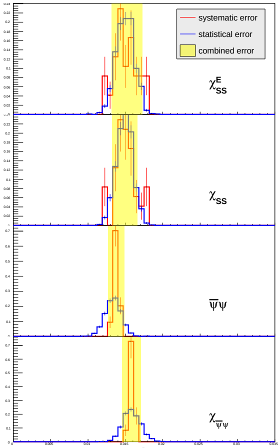


Extrap. with $Nt = 10, 12, 16$



Comparison for different observables

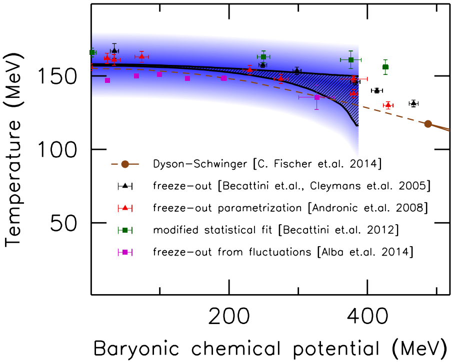


χ^E_{SS}: $\langle n_S \rangle = 0$ and $0.5\langle B \rangle = \langle Q \rangle$

χ_{SS}: $\langle n_S \rangle = 0$ and $0.4\langle B \rangle = \langle Q \rangle$

T_c extrapolation

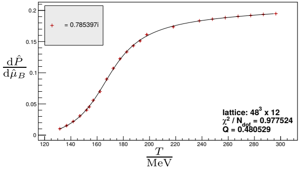
Determining $T_c(\mu_B)$ by solving the equation $\frac{T_c(\mu_B)}{T_c(0)} = C_i\left(-\frac{\mu_B^2}{T_c^2(\mu)}\right)$.



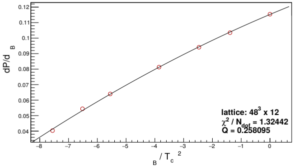
$$C_0(x) = 1 + ax$$
$$C_1(x) = 1 + ax + bx^2$$
$$C_2(x) = \frac{1 + ax}{1 + bx}$$
$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Overview over the Analysis

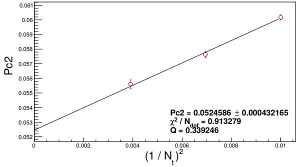
1. Do the simulations at $\langle n_s \rangle \approx 0$
2. Extrapolate to $\langle n_s \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$
3. Make a fit in the T direction



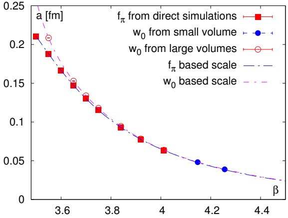
4. Determine everything you need for the observables
5. Make a fit in the μ_B direction



6. Make a fit in the $\frac{1}{N_t^2}$ direction
7. Determine the observables



Simulation details



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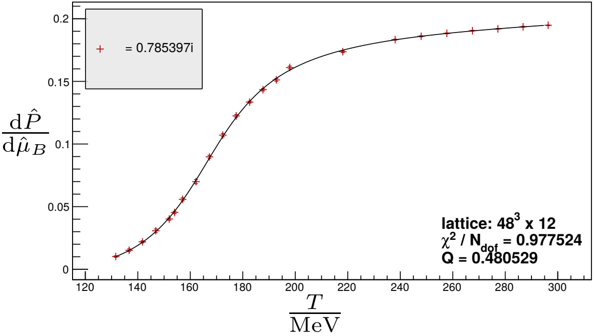
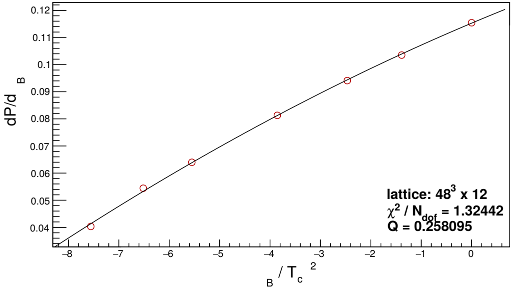
- Lattice sizes: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5,$ 6, 6.5 and 7
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Fit in the T direction



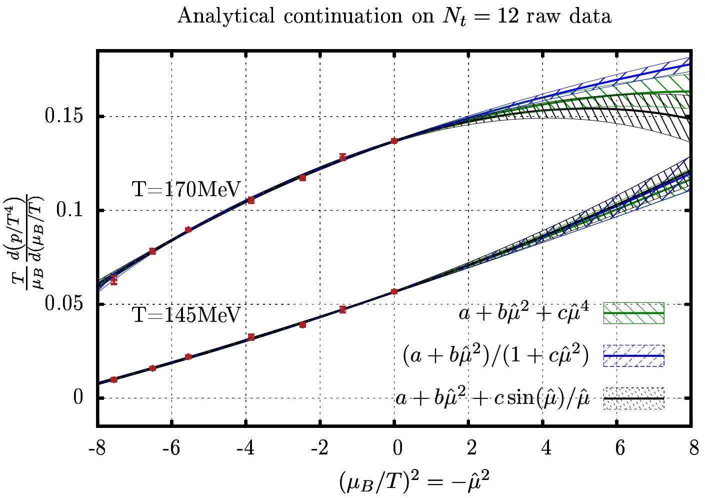
$$
\begin{aligned}
A_1(T) &= a + bT + c/T + d\arctan(e(T-f)) \\
A_2(T) &= a + bT + c/T + d/(1 + e(T-f)^g)^{1/g}, \\
A_3(T) &= a + bT + cT^2 + d\arctan(e(T-f)) \\
A_4(T) &= a + bT + cT^2 + d/(1 + e(T-f)^g)^{1/g}.
\end{aligned}
$$

Fit in the μ_B direction



$$
\begin{aligned}
B_1(\hat{\mu}) &= a + b\hat{\mu}^2 + c\hat{\mu}^4 \\
B_2(\hat{\mu}) &= (a + b\hat{\mu}^2)/(1 + c\hat{\mu}^2) \\
B_3(\hat{\mu}) &= a + b\hat{\mu}^2 + c\sin(\hat{\mu})/\hat{\mu}
\end{aligned}
$$

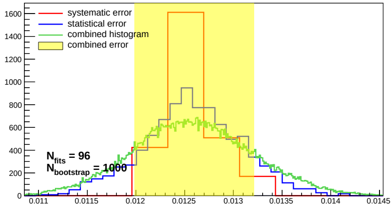
Extrapolation from different fit functions



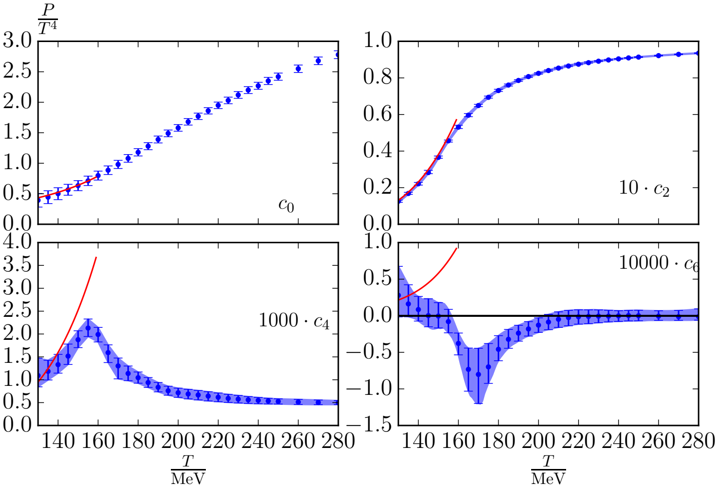
Error estimation

- Statistical error:
  Bootstrap method
- Systematic error:
  Using different way of analysis, combining them in a histogram:
  - 4 fit functions for the T direction
  - 3 fit functions in the μ_B direction
  - Doing continuum extrapolation and μ_B-fit in one or two steps
  - 2 methods of scale setting: f_π and w_0
  - 2 temperatures from where we use the extrapolated data

  This adds up to 96 ways of analysis

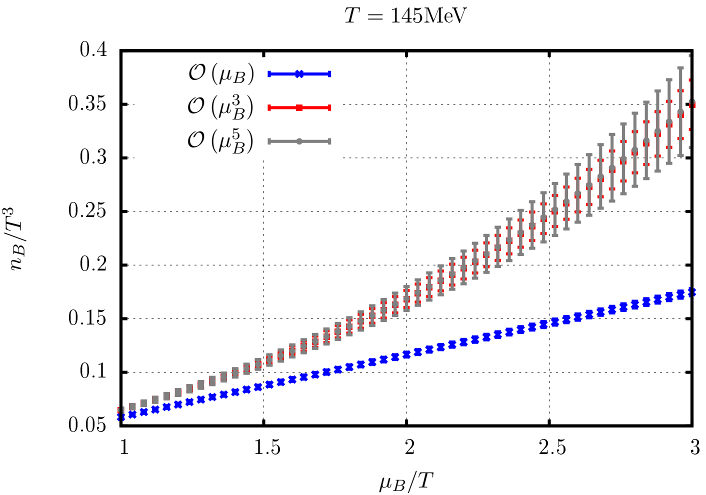


Taylor coefficients

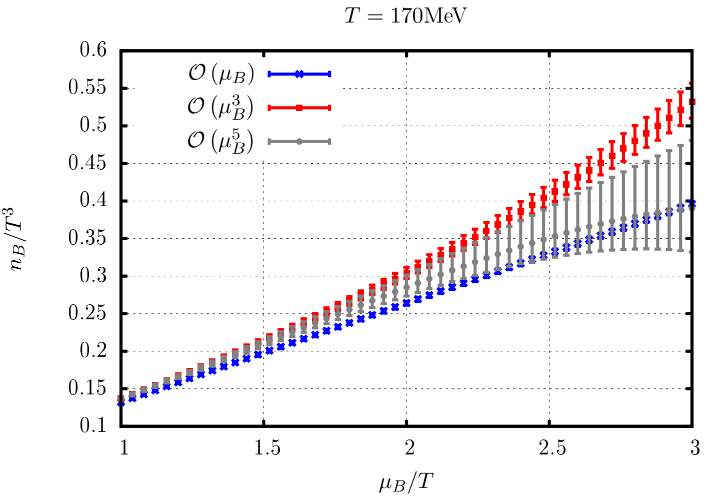


The Taylor coefficients of $\frac{P}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 + c_4 \left(\frac{\mu_B}{T}\right)^4 + c_6 \left(\frac{\mu_B}{T}\right)^6$

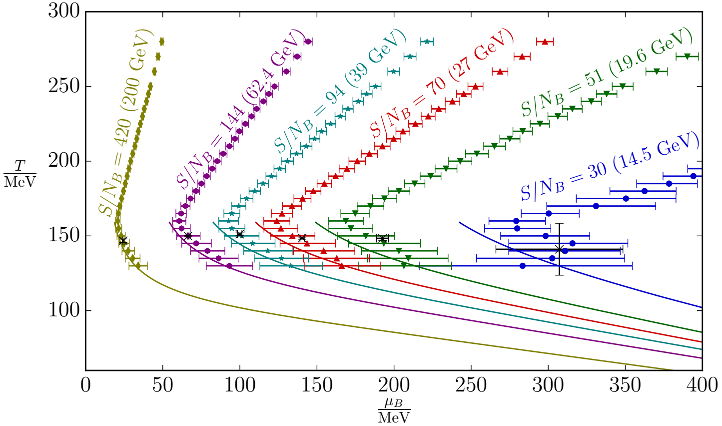
Influence of different orders



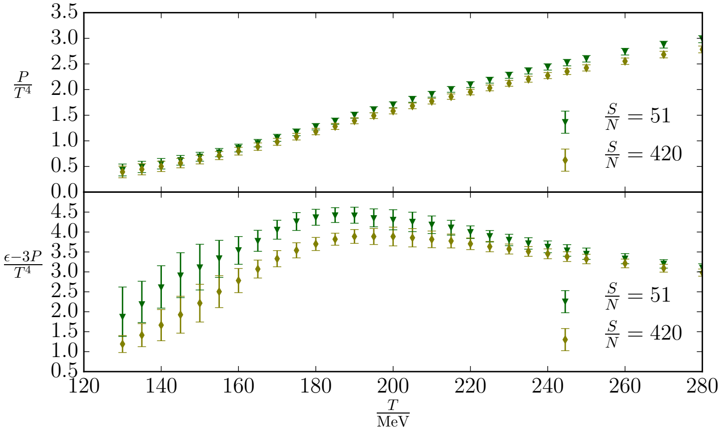
Influence of different orders



Trajectories



Equation of state



Conclusion

- Lattice can investigate the phase diagram at small chemical potential (up to $\approx$ 300 MeV) by analytical continuation via
  1. Taylor expansion method
  2. Simulations at imaginary μ
- Results for the transition temperature
- Results for the equation of state up to order μ^6
- This approach only works up to a critical point
- Other methods have to be found beyond that $\longrightarrow$ two talks this afternoon